System Parameter Identification of Quadrotor

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Final Report
15:10 - 18:10, June 15 (Tuesday), 2021
Curriculum on Applied System Identification
Department of Engineering Science
National Cheng Kung University
Tainan, Taiwan
February 22 - June 22, 2021

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Outline

Outline

Section 1

Introduction

- Motivation
- Main Difficulties for Estimating Quadrotor's Parameters and Solutions

Motivation: Why do we identify the parameters of the system?

Motivation

- Identifying the system parameters is useful for the control design.
- Also, it can be applied in the redesign of dynamic configuration for obtaining more better dynamic properties.

Outline

What are the parameters to be identified?

- Governing equation:

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\mathbf{J}\boldsymbol{\omega} = \mathbf{M}_G$$

- The following parameters should be identified:

$$\mathbf{J} = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix}$$

- System inputs: $\mathbf{M}_G = \begin{bmatrix} M_{Gx} & M_{Gy} & M_{Gz} \end{bmatrix}^T$
- System outputs: $\boldsymbol{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$

Least-Squares Formulation for Inertia Estimation (1/2)

- Least-Squares formulation

$$\underbrace{\mathbf{M}_G(t)}_{\mathbf{y}_{3\times 1}} = \underbrace{\begin{bmatrix} \boldsymbol{\varphi}_1(t) & \boldsymbol{\varphi}_2'(t) & \boldsymbol{\varphi}_3'(t) & \boldsymbol{\varphi}_5(t) & \boldsymbol{\varphi}_6'(t) & \boldsymbol{\varphi}_9(t) \end{bmatrix}}_{\boldsymbol{\varphi}'_{3\times 6}} \underbrace{\begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{xz} \\ J_{yy} \\ J_{yz} \\ J_{zz} \end{bmatrix}}_{\mathbf{x}'_{6\times 1}} \tag{1}$$

where

$$\begin{aligned} \boldsymbol{\varphi}_2'(t) &= \boldsymbol{\varphi}_2(t) + \boldsymbol{\varphi}_4(t) \\ \boldsymbol{\varphi}_3'(t) &= \boldsymbol{\varphi}_3(t) + \boldsymbol{\varphi}_7(t) \\ \boldsymbol{\varphi}_6'(t) &= \boldsymbol{\varphi}_6(t) + \boldsymbol{\varphi}_8(t) \end{aligned} \tag{2}$$

and

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \boldsymbol{\varphi}_3 & \boldsymbol{\varphi}_4 & \boldsymbol{\varphi}_5 & \boldsymbol{\varphi}_6 & \boldsymbol{\varphi}_7 & \boldsymbol{\varphi}_8 & \boldsymbol{\varphi}_9 \end{bmatrix}_{3\times 9} \\ &= \begin{bmatrix} \dot{\omega}_x \mathbf{I}_3 + \omega_x \boldsymbol{\omega}^\times & \dot{\omega}_y \mathbf{I}_3 + \omega_y \boldsymbol{\omega}^\times & \dot{\omega}_z \mathbf{I}_3 + \omega_z \boldsymbol{\omega}^\times \end{bmatrix}_{3\times 9} \end{aligned} \tag{3}$$

Least-Squares Formulation for Inertia Estimation (2/2)

- Given $t = T, 2T, \cdots, nT$ where T is the sample interval, the equation for inertia estimation becomes

$$\begin{bmatrix} \mathbf{M}_G(T) \\ \mathbf{M}_G(2T) \\ \vdots \\ \mathbf{M}_G(nT) \end{bmatrix} = \begin{bmatrix} \varphi_1(T) & \varphi_2'(T) & \varphi_3'(T) & \varphi_5(T) & \varphi_6'(T) & \varphi_9(T) \\ \varphi_1(2T) & \varphi_2'(2T) & \varphi_3'(2T) & \varphi_5(2T) & \varphi_6'(2T) & \varphi_9(2T) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_1(nT) & \varphi_2'(nT) & \varphi_3'(nT) & \varphi_5(nT) & \varphi_6'(nT) & \varphi_9(nT) \end{bmatrix} \begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{xz} \\ J_{yy} \\ J_{yz} \\ J_{zz} \end{bmatrix} \tag{4}$$

- The least-squares solution for $n \geq 6$ is

$$\begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{xz} \\ J_{yy} \\ J_{yz} \\ J_{zz} \end{bmatrix} = \begin{bmatrix} \varphi_1(T) & \varphi_2'(T) & \varphi_3'(T) & \varphi_5(T) & \varphi_6'(T) & \varphi_9(T) \\ \varphi_1(2T) & \varphi_2'(2T) & \varphi_3'(2T) & \varphi_5(2T) & \varphi_6'(2T) & \varphi_9(2T) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_1(nT) & \varphi_2'(nT) & \varphi_3'(nT) & \varphi_5(nT) & \varphi_6'(nT) & \varphi_9(nT) \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{M}_G(T) \\ \mathbf{M}_G(2T) \\ \vdots \\ \mathbf{M}_G(nT) \end{bmatrix} \tag{5}$$

Main Difficulties for Estimating Quadrotor's Parameters

Main Difficulties

- In practical scenarios, flight vehicles' unstable response may be induced for an open-loop excitation input.
- The angular acceleration $[\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z]^T$ can not be accurately obtained in general.

Solutions

- To obtain a stable flight trajectory, the closed-loop excitation strategy is presented.
- The Observer/Kalman Filter Identification (OKID) is applied to estimate velocity and acceleration.
- A straightforward method to avoid velocity differentiation amplifying measured noise, namely the integral operator method, is used.

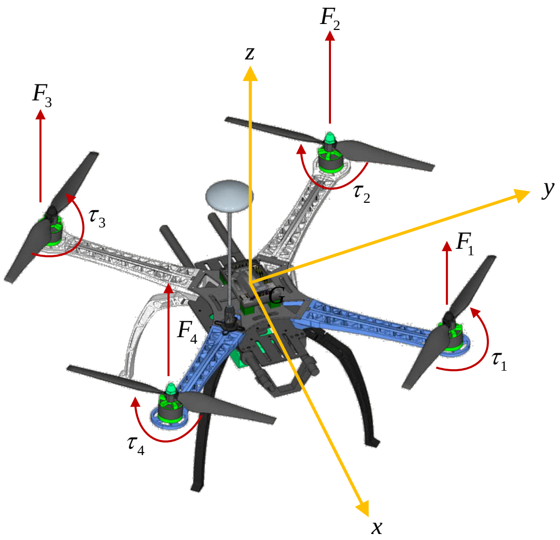
Outline

Section 3

Dynamics Modeling

- Dynamic Configuration and Mathematical Model
- Configuration of Control and System Identification

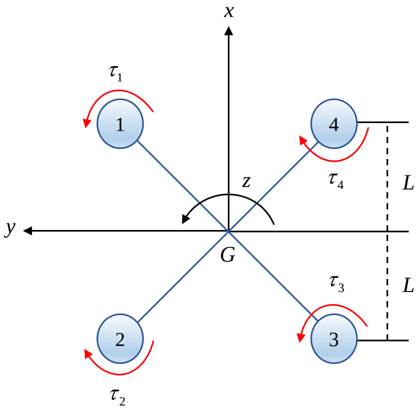
Dynamic Configuration of Quadrotor



[1]

[1] Graphic source: `https://3dwarehouse.sketchup.com/model/u785a963c-b533-46e7-8c19-20482ccda5b8/S500-Quadcopter?hl=en`

Forces and Torques Generated by Actuators



The force F_{Gz} and torques $[M_{Gx}, M_{Gy}, M_{Gz}]$ expressed as F_i and τ_i generated by actuators:

$$\begin{aligned} F_{Gz} &= F_1 + F_2 + F_3 + F_4 \\ M_{Gx} &= F_1 L + F_2 L - F3L - F_4 L \\ M_{Gy} &= -F_1 L + F_2 L + F3L - F_4 L \\ M_{Gz} &= \tau_1 - \tau_2 + \tau_3 - \tau_4 \end{aligned} \tag{6}$$

Mathematical Model of Quadrotor

- Translation Dynamics

$$m\ddot{\mathbf{P}} = \mathbf{W} + \mathbf{R}_B^G\left(q_0, \mathbf{q}\right)\mathbf{F}_a \quad \rightarrow \quad \begin{aligned} m\ddot{X} &= 2(q_1q_3 + q_0q_2)F_{Gz} \\ m\ddot{Y} &= 2(q_2q_3 - q_0q_1)F_{Gz} \\ m\ddot{Z} &= \left(q_0^2 - q_1^2 - q_2^2 + q_3^2\right)F_{Gz} - mg \end{aligned} \tag{7}$$

where F_{Gz} is the control force.

- Rotation Dynamics

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T\boldsymbol{\omega} \tag{8}$$

$$\dot{\mathbf{q}} = \frac{1}{2}\left(q_0\mathbf{I}_3 + \mathbf{q}^\times\right)\boldsymbol{\omega} \tag{9}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} = \mathbf{M}_G - \boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} \tag{10}$$

where $\mathbf{M}_G = [M_{Gx}, M_{Gy}, M_{Gz}]^T$ is the control torque.

Flight Controllers

- Altitude Controller

$$F_{Gz} = \frac{m}{r_{33}} \left[\ddot{Z}_d + g - k_{p,z} \left(Z - Z_d \right) - k_{d,z} \left(\dot{Z} - \dot{Z}_d \right) \right] \tag{11}$$

  where $r_{33} = q_0^2 - q_1^2 - q_2^2 + q_3^2$.

- Attitude Controller

$$\mathbf{M}_{G,u} = -\mathbf{K}_p \mathbf{q} - \mathbf{K}_v \boldsymbol{\omega} \tag{12}$$

- Total Input Torque

$$\mathbf{M}_G = \mathbf{M}_{G,u} + \mathbf{r} \tag{13}$$

  where $\mathbf{r}$ is the excitation input.

Actuator Dynamics (Rotor Dynamics)

- Actuator Configuration
  - Thrusts and Torques generated by rotors

$$\begin{aligned} F_i &= C_T \Omega_i^2 \\ \tau_i &= C_M \Omega_i^2, \quad i = 1, 2, 3, 4. \end{aligned} \tag{14}$$

  - Force transmission matrix

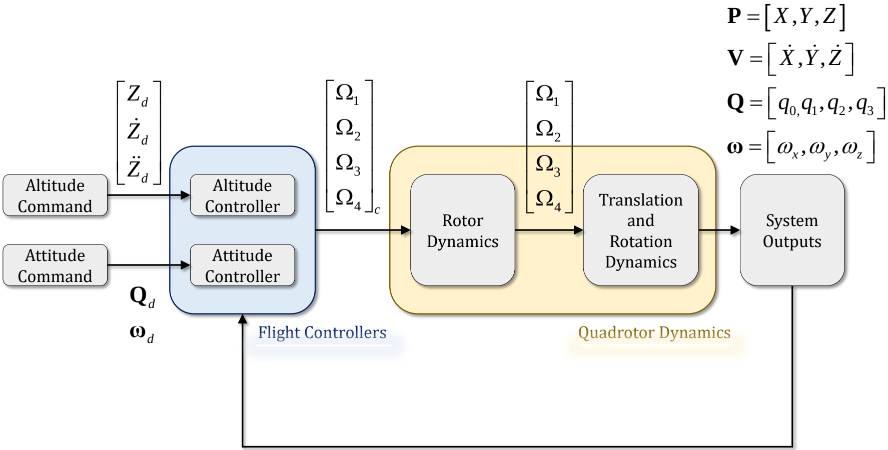
$$\begin{bmatrix} F_{Gz} \\ M_{Gx} \\ M_{Gy} \\ M_{Gz} \end{bmatrix} = \begin{bmatrix} C_T & C_T & C_T & C_T \\ C_T L & C_T L & -C_T L & -C_T L \\ -C_T L & C_T L & C_T L & -C_T L \\ C_M & -C_M & C_M & -C_M \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{15}$$

- Actuator dynamics (For i-th rotor)

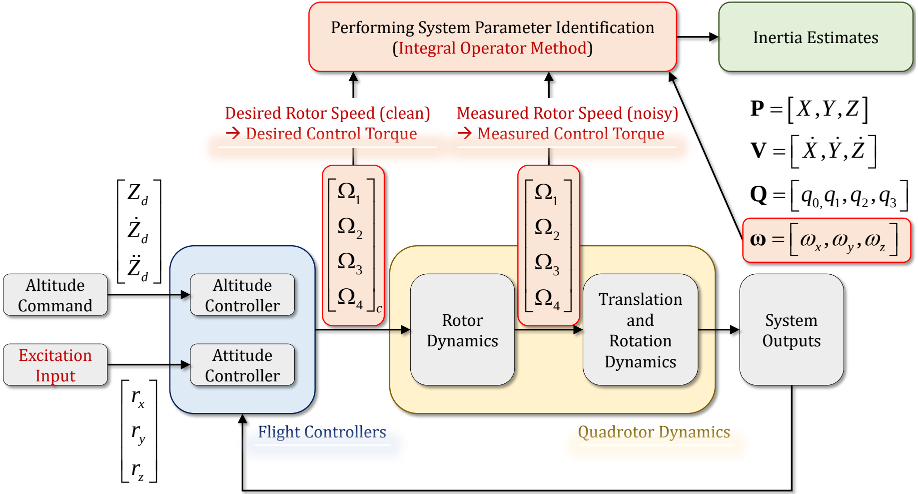
$$G_a(s) = \frac{\Omega_r(s)}{\Omega_c(s)} = \frac{T_2}{T_1 s + 1} \tag{16}$$

where $\Omega_r(s) = \mathcal{L}\{\Omega_c(t)\}$. $T_1 = 0.001$, $T_2 = 0.99$ are considered in the following simulations.

Control Configuration of Quadrotor



How to Perform the Inertia Estimation?



Outline

Section 4

Simulation Background and I/O Data

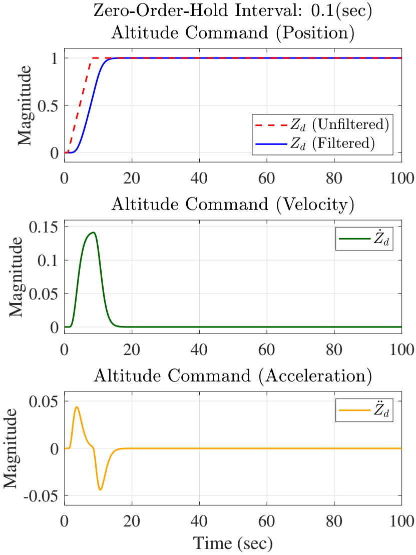
- Simulation Background
- I/O Data

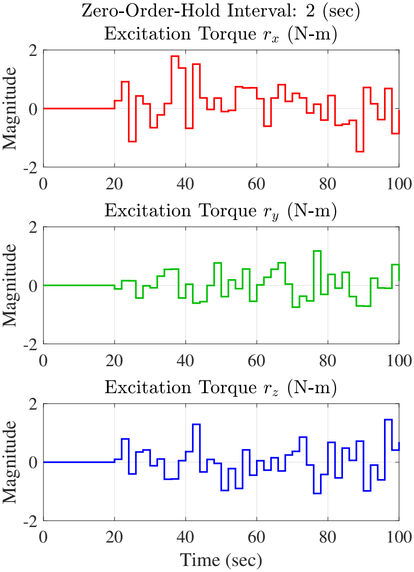
Simulation Background

Simulation Background:

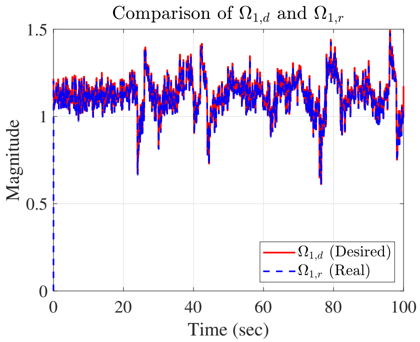
- Sample Step Size of RK4: 0.0005 (sec)
- Sample Interval of Output Data: 0.01 (sec)
- Zero-Order-Hold Interval of Control Inputs: 0.1 (sec)
- Number of Data: 7001 Points
- Control Gains: $\mathbf{K}_p = \mathrm{diag}([6, 6, 6])$; $\mathbf{K}_v = \mathrm{diag}([5, 5, 5])$
- Excitation Inputs $\mathbf{r}$: PRBS sequences ($T_{zoh} = 2$ (sec)).
- $C_T, C_M, L, m, g, T_1, T_2 \quad \rightarrow \quad$ Pre-identified parameters.

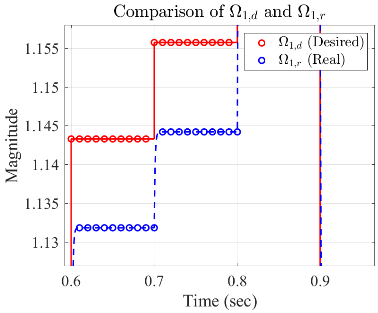
Reference Altitude Command and Excitation Input



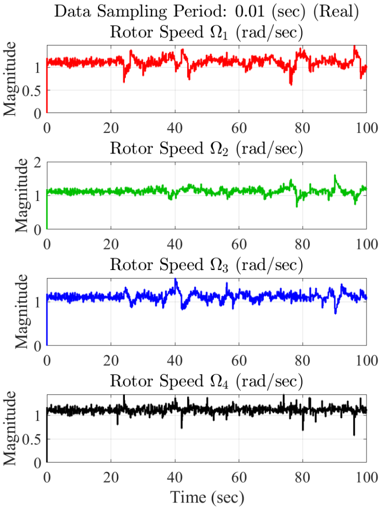


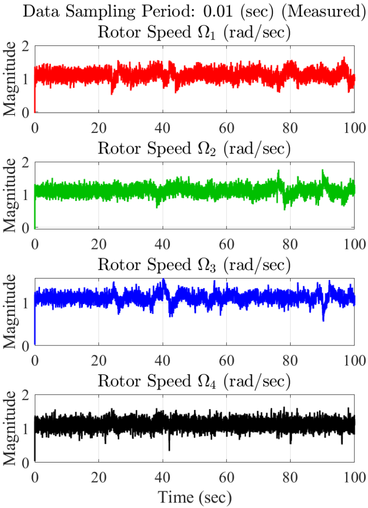
Desired Rotor Speed and Real Rotor Speed



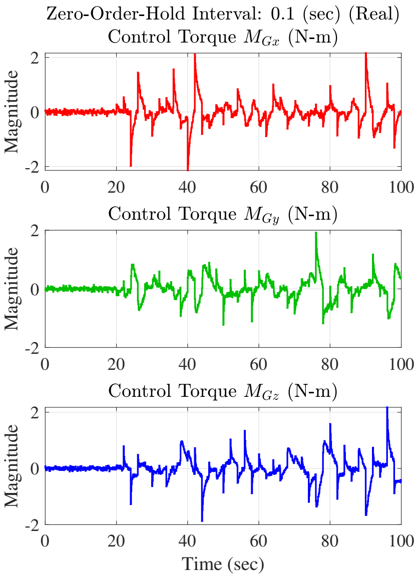


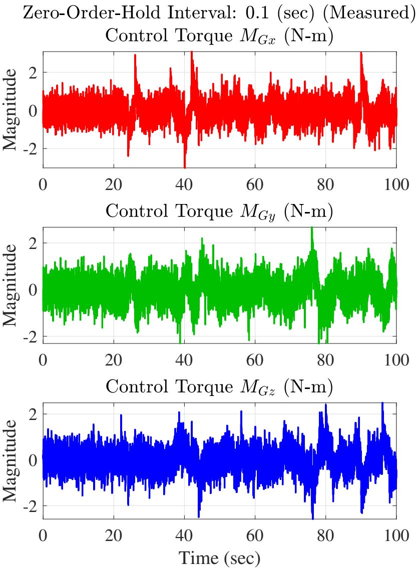
Real Rotor Speed and Measured Rotor Speed



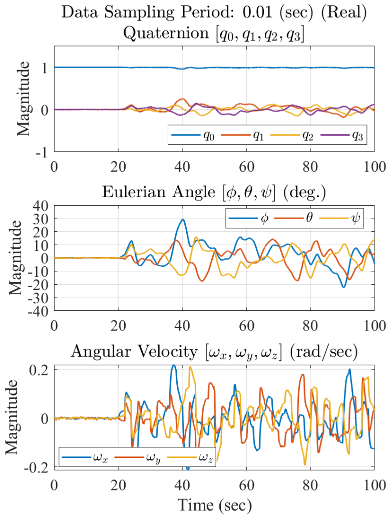


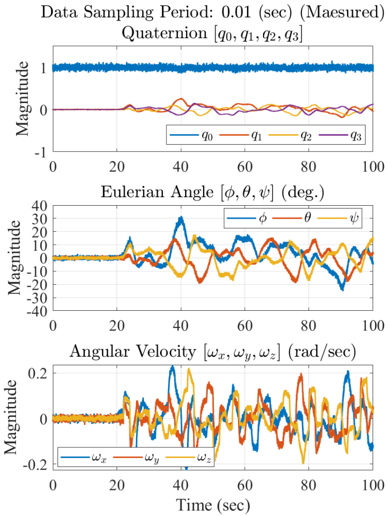
Real Torque and Measured Torque



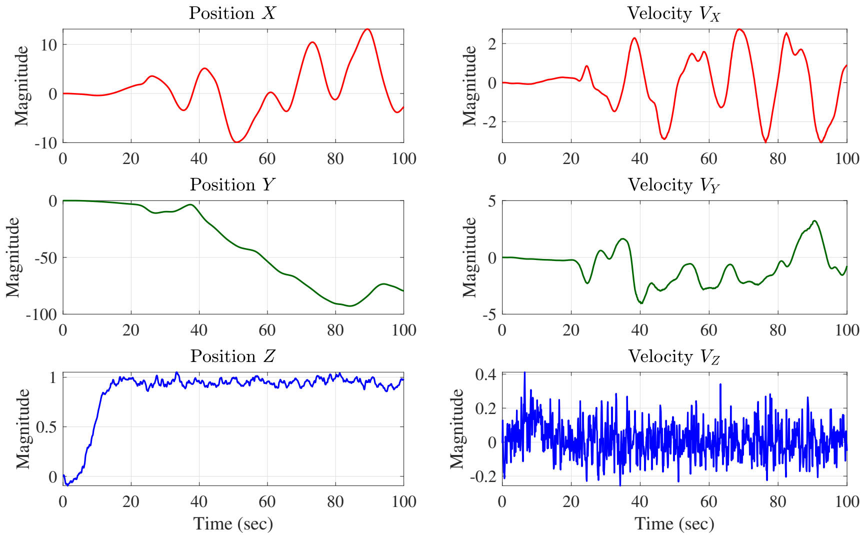


Real Output and Measured Output





Position and Velocity



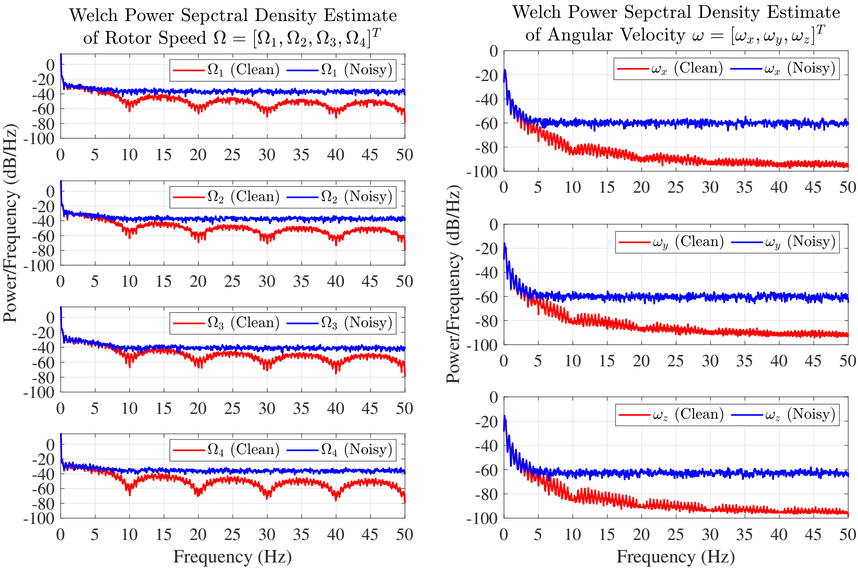
Outline

Section 5

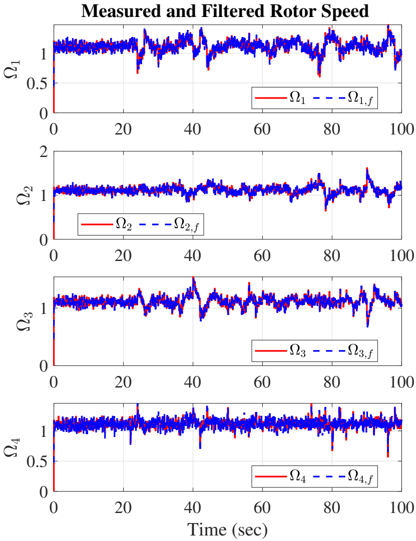
Signal Processing

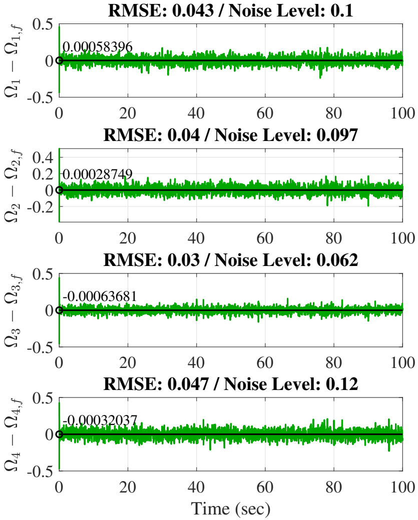
- Spectrum Analysis
- Signal Processing for I/O Data

Spectrum Analysis

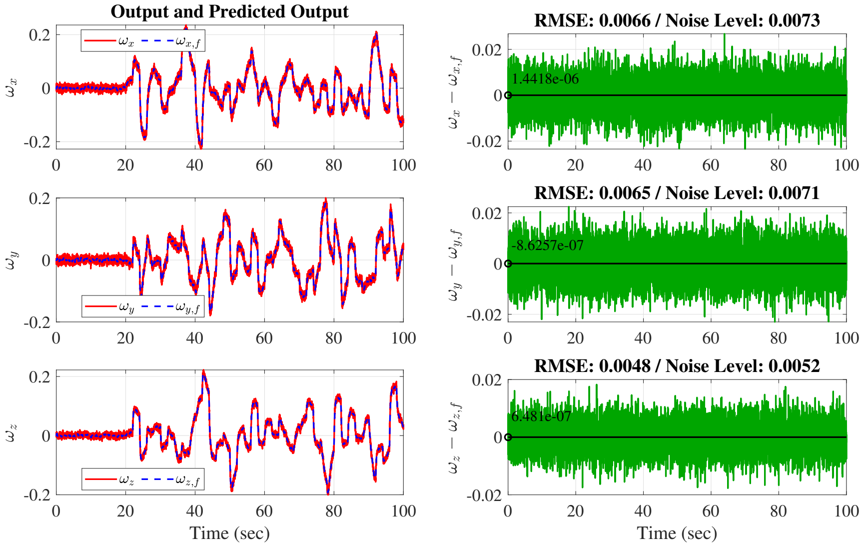


Measured Rotor Speed and Filtered Rotor Speed

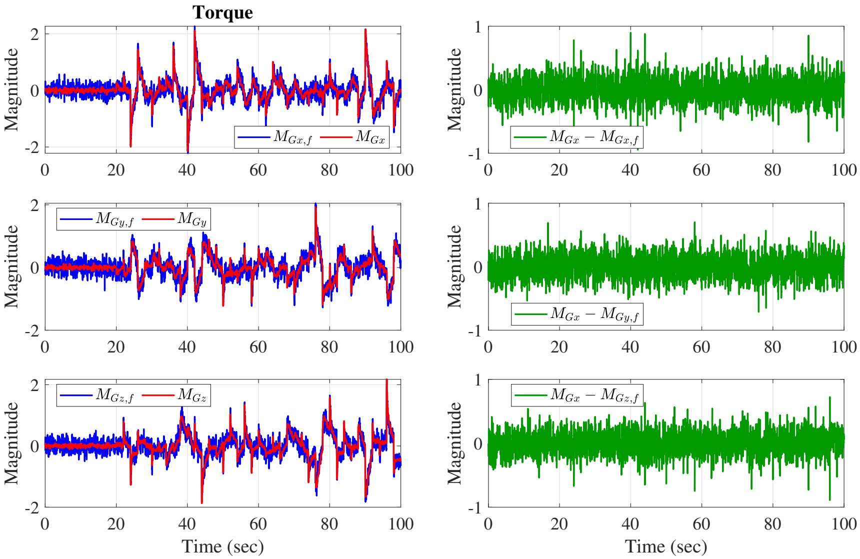




Measured Velocity and Filtered Velocity



Input Torque Reconstruction



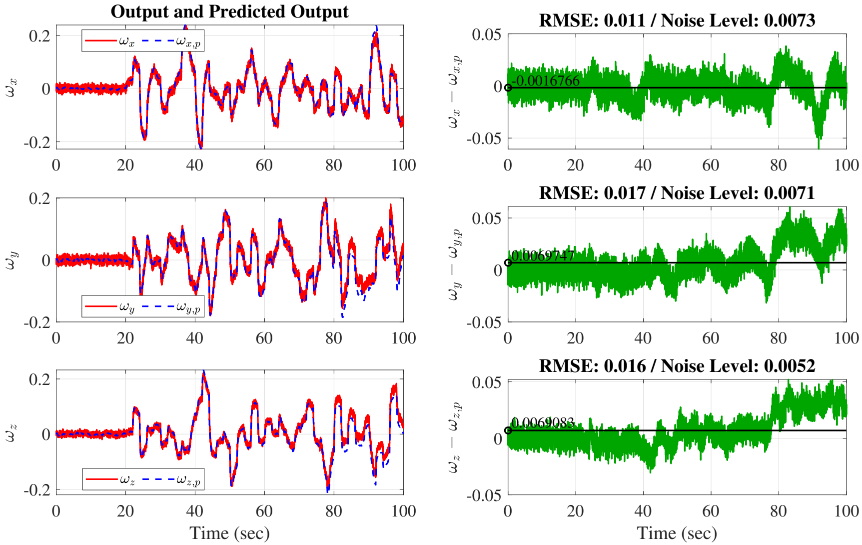
Outline

Section 6

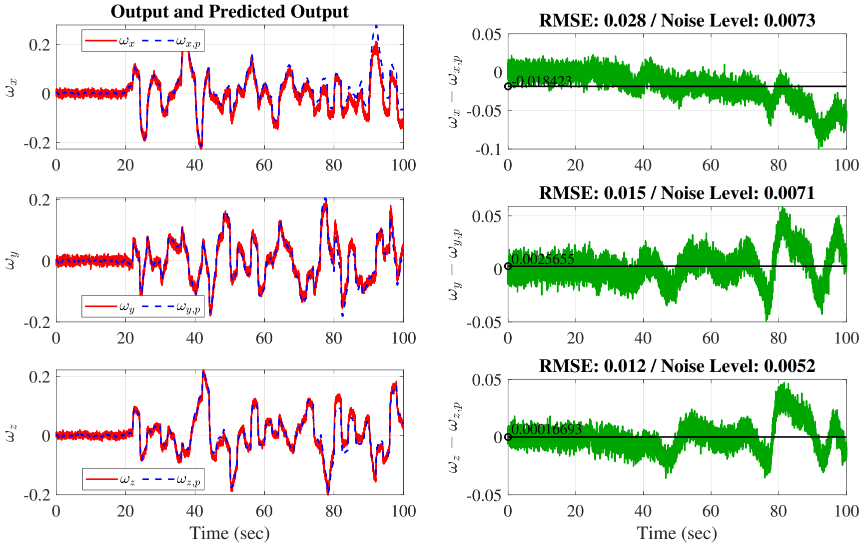
Parameter Identification via Integral Method

- A. Using Filtered Input and Output Data
- B. Using Desired Input and Filtered Output Data

A. Using Filtered Input and Output Data ($\lambda = 1$)



B. Using Desired Input and Filtered Output ($\lambda = 1$)



Comparison of Parameter Estimates

Table: Comparison of Real and Identified Parameters (Case A and Case B).

Para.	Real	Case A	Case B
J_{xx}	5.0000	5.0863 (1.73)	5.2316 (4.62)
J_{xy}	-2.0000	-2.0506 (2.53)	-2.1054 (5.27)
J_{xz}	-1.0000	-0.9101 (8.99)	-0.9905 (0.95)
J_{yy}	6.0000	6.1619 (2.70)	6.2360 (3.93)
J_{yz}	-4.0000	-4.0039 (0.01)	-4.0621 (1.55)
J_{zz}	7.0000	6.6614 (4.84)	6.9693 (0.49)

Table: Root-mean-squares error of predicted error for Case A and Case B.

RMSE	Case A	Case B
$e_{rms,x} = \omega_x - \omega_{x,p}$	0.011	0.028
$e_{rms,y} = \omega_y - \omega_{y,p}$	0.017	0.015
$e_{rms,z} = \omega_z - \omega_{z,p}$	0.016	0.012

Outline

Section 7

Velocity and Acceleration Estimation via OKID

- Observer Equation
- Velocity and Acceleration Estimation via Okid
- Simulation Results

Observer Equation of Okid (1/2)

- Input and Output data

$$\begin{aligned} &\text{Angular velocity (Filtered)} \qquad && \mathbf{y} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T \\ &\text{Desired control torque} && \mathbf{u} = \begin{bmatrix} M_{Gx} & M_{Gy} & M_{Gz} \end{bmatrix}^T \end{aligned} \tag{17}$$

- Using OKID gives the following observer equation

$$\begin{aligned} \tilde{\mathbf{x}}(k+1) &= \tilde{\mathbf{A}}\tilde{\mathbf{x}}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) + \tilde{\mathbf{G}}\left[\tilde{\mathbf{y}}(k) - \mathbf{y}(k)\right], \quad \tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0 \\ \tilde{\mathbf{y}}(k) &= \tilde{\mathbf{C}}\tilde{\mathbf{x}}(k) + \tilde{\mathbf{D}}\mathbf{u}(k) \end{aligned} \tag{18}$$

where

$$\begin{aligned} &\tilde{\mathbf{A}}: \quad 3p \times 3p \qquad && \tilde{\mathbf{B}}: \quad 3p \times 3 && \\ &\tilde{\mathbf{C}}: \quad 3 \times 3p && \tilde{\mathbf{D}}: \quad 3 \times 3 \qquad && \tilde{\mathbf{G}}: \quad 3p \times 3 \end{aligned} \tag{19}$$

and p is the order of ARX model.

- The force transmission matrix $\tilde{\mathbf{D}} = \mathbf{0}$ for velocity and acceleration measurements.

Observer Equation of Okid (2/2)

- Let $p = 1$. The system matrices of observer equation are

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0.9999 & 0.0002 & -0.0004 \\ -0.0003 & 1.0002 & -0.0003 \\ -0.0002 & 0.0002 & 0.9997 \end{bmatrix}; \quad \tilde{\mathbf{B}} = \begin{bmatrix} 0.0027 & 0.0018 & 0.0014 \\ 0.0017 & 0.0034 & 0.0023 \\ 0.0013 & 0.0020 & 0.0028 \end{bmatrix};$$

$$\tilde{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \tilde{\mathbf{G}} = \begin{bmatrix} -0.9999 & -0.0002 & 0.0004 \\ 0.0003 & -1.0002 & 0.0003 \\ 0.0002 & -0.0002 & -0.9997 \end{bmatrix} \tag{20}$$

- The eigenvalues of $\tilde{\mathbf{A}}$ are

$$\lambda(\tilde{\mathbf{A}}) = \begin{bmatrix} 1.0001 + 0.0001i \\ 1.0001 - 0.0001i \\ 0.9997 \end{bmatrix} \tag{21}$$

- The estimated initial condition is

$$\tilde{\mathbf{x}}_0 = \begin{bmatrix} 0.0273 \\ -0.0073 \\ 0.0291 \end{bmatrix}; \quad \mathbf{y}(0) = \begin{bmatrix} 0.0273 \\ -0.0073 \\ 0.0291 \end{bmatrix} \tag{22}$$

Velocity and Acceleration Estimations via Okid (1/2)

- The velocity estimates are the outputs of observer equation.

$$\begin{bmatrix} \hat{\omega}_x(k) & \hat{\omega}_y(k) & \hat{\omega}_z(k) \end{bmatrix} = \tilde{\mathbf{y}}(k) \tag{23}$$

- The acceleration estimates can be determined by the following steps:
  - Convert the discrete-time system matrices $(\tilde{\mathbf{A}}, \tilde{\mathbf{B}})$ into continuous-time system matrices $(\tilde{\mathbf{A}}_c, \tilde{\mathbf{B}}_c)$.

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \tilde{\mathbf{A}}_c \mathbf{x}(t) + \tilde{\mathbf{B}}_c \mathbf{u}(t) \\ \tilde{\mathbf{y}}(t) &= \tilde{\mathbf{C}} \tilde{\mathbf{x}}(t) \end{aligned} \tag{24}$$

  - Differentiating $\tilde{\mathbf{y}}(t)$ gives

$$\dot{\tilde{\mathbf{y}}}(t) = \tilde{\mathbf{C}} \dot{\tilde{\mathbf{x}}}(t) \tag{25}$$

$$= \tilde{\mathbf{C}} \tilde{\mathbf{A}}_c \tilde{\mathbf{x}}(t) + \tilde{\mathbf{C}} \tilde{\mathbf{B}}_c \mathbf{u}(t) \tag{26}$$

  - Given $t = T, 2T, \cdots, kT$ where T is the sample interval, the acceleration estimates in sample index are

$$\begin{aligned} \begin{bmatrix} \dot{\hat{\omega}}_x(k) & \dot{\hat{\omega}}_y(k) & \dot{\hat{\omega}}_z(k) \end{bmatrix} &= \dot{\tilde{\mathbf{y}}}(k) \\ \dot{\tilde{\mathbf{y}}}(k) &= \tilde{\mathbf{C}} \tilde{\mathbf{A}}_c \tilde{\mathbf{x}}(k) + \tilde{\mathbf{C}} \tilde{\mathbf{B}}_c \mathbf{u}(k) \end{aligned} \tag{27}$$

  - where $k = 1, 2, \cdots, n$. Notice that the estimated states $\tilde{\mathbf{x}}(k)$ are computed from observer equation (18).

Velocity and Acceleration Estimations via Okid (2/2)

- Conclusively, we can use the measured info to estimate velocity and acceleration:

$$
\begin{aligned}
\tilde{\mathbf{x}}(k+1) &= \tilde{\mathbf{A}}\tilde{\mathbf{x}}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) + \tilde{\mathbf{G}}\left[\tilde{\mathbf{y}}(k) - \mathbf{y}(k)\right], \quad \tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0 \\
\tilde{\mathbf{y}}(k) &= \tilde{\mathbf{C}}\tilde{\mathbf{x}}(k) \\
\dot{\tilde{\mathbf{y}}}(k) &= \tilde{\mathbf{C}}\tilde{\mathbf{A}}_c\tilde{\mathbf{x}}(k) + \tilde{\mathbf{C}}\tilde{\mathbf{B}}_c\mathbf{u}(k) \\
\hat{\boldsymbol{\omega}}(k) &= \tilde{\mathbf{y}}(k) \\
\dot{\hat{\boldsymbol{\omega}}}(k) &= \dot{\tilde{\mathbf{y}}}(k)
\end{aligned}
\tag{28}
$$

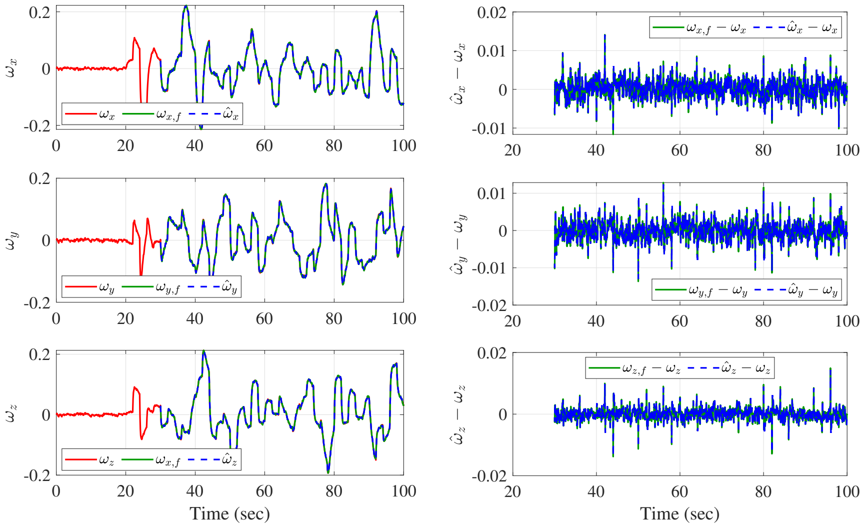
where

$$
\tilde{\mathbf{A}}_c = \begin{bmatrix} -0.0071 & 0.0217 & -0.0383 \\ -0.0287 & 0.0203 & -0.0285 \\ -0.0222 & 0.0158 & -0.0305 \end{bmatrix}; \quad \tilde{\mathbf{B}}_c = \begin{bmatrix} 0.2738 & 0.1774 & 0.1394 \\ 0.1741 & 0.3388 & 0.2276 \\ 0.1315 & 0.2049 & 0.2830 \end{bmatrix} \tag{29}
$$

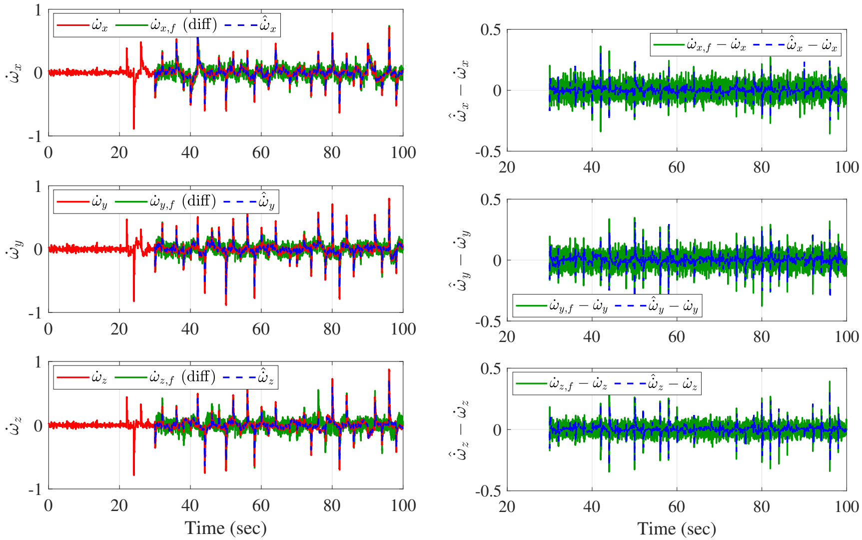
- The eigenvalues of $\tilde{\mathbf{A}}_c$ are

$$
\lambda(\tilde{\mathbf{A}}_c) = \begin{bmatrix} 0.0071 + 0.0087i \\ 0.0071 - 0.0087i \\ -0.0315 \end{bmatrix} \tag{30}
$$

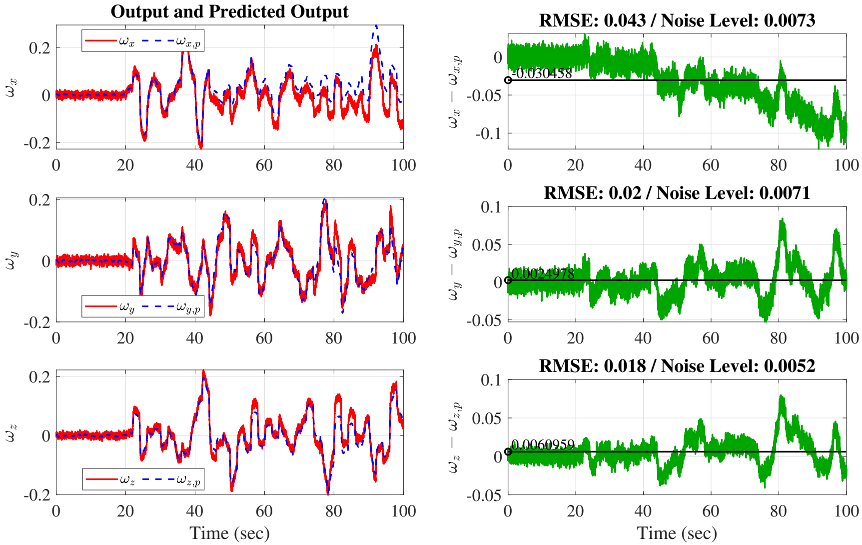
Comparison of Velocity and Estimated Velocity



Comparison of Acceleration and Estimated Acceleration



Output Prediction Using Identified Parameters



Comparison of Parameter Estimates

Table: Comparison of real and identified parameters (Integral Method, OKID and Diff.).

Para.	Real	Integral Method (A)	OKID	Diff.
J_{xx}	5.0000	5.0863 (1.73)	5.4292 (8.58)	3.7611 (24.78)
J_{xy}	-2.0000	-2.0506 (2.53)	-2.1101 (5.51)	-1.06 (46.76)
J_{xz}	-1.0000	-0.9101 (8.99)	-0.9690 (3.10)	-0.94 (6.27)
J_{yy}	6.0000	6.1619 (2.70)	6.3449 (5.75)	3.5322 (41.43)
J_{yz}	-4.0000	-4.0039 (0.10)	-3.9658 (0.85)	-2.0520 (48.70)
J_{zz}	7.0000	6.6614 (4.84)	7.2065 (2.95)	4.8985 (30.02)

Table: Root-mean-squares error of predicted error for different approaches (Integral Method and OKID).

RMSE	Integral Method	OKID
$e_{rms,x} = \omega_x - \omega_{x,p}$	0.011	0.043
$e_{rms,y} = \omega_y - \omega_{y,p}$	0.017	0.020
$e_{rms,z} = \omega_z - \omega_{z,p}$	0.016	0.018

Outline

Section 8

Conclusion

Conclusion

Cluclusion

- The least-squares formulation for estimating the parameters of the Euler equation is proposed.
- A closed-loop excitation strategy is presented for obtaining the stable flight trajectory.
- The integral operator method is introduced to perform the parameter identification.
- The velocity and acceleration estimation via OKID is conducted.
- The simulation results reveal that using OKID gives a better parameter estimating performance than numerical differentiation.
- The integral method provides the best parameter estimation in three approaches.

Outline

Section 9

Future Work

Future Work

Future Work

- Experimental Validation.